NOISE ESTIMATION IN DIGITAL IMAGES USING FUZZY PROCESSING

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ABSTRACT

The noise estimation is an important issue in image processing because it is a fundamental step in many algorithms for the noise suppression and then for the image restoration. In literature many approaches have been presented in order to obtain good results. This paper presents a novel method suitable to get a good estimation if the type of the noise distribution is known. In particular the algorithm provides the variance of noise distribution and the proof that distribution itself matches the foreseen one. The algorithm has been tested on different images affected by gaussian noise and the simulations show results better than those obtained with other approaches.

1. INTRODUCTION

In literature many approaches have been presented in order to estimate the type of the noise and its distribution density as better as possible. A particular and interesting case regards the images.
The importance of the estimation process is due to the necessity of tuning the filtering parameters [1]. Different setting of these features may generate very different filter actions, successfully or not. So the noise features estimation may help knowing the noise behavior and eliminating it. In the first part of this paper we will show various kind of noise estimation, above all noise variance estimation; then we will describe a novel approach using the χ^2 parameter.

2. VARIOUS KIND OF NOISE

A typical imaging system consists of an image formation system, a detector and a recorder [2]. A general model for such systems can be expressed as

$$v(x,y) = g[w(x,y)] + \eta(x,y) \quad (1)$$

$$w(x,y) = \int\int_{-\infty}^{\infty} h(x,y;x',y')u(x',y')dx'dy' \quad (2)$$

$$\eta(x,y) = f[g(w(x,y))]\eta_1(x,y) + \eta_2(x,y) \quad (3)$$

where $u(x,y)$ represents the object (original image) and $v(x,y)$ is the observed image. The function $h(x,y;x',y')$ represents the impulse response of the formation process and the functions $f(\cdot), g(\cdot)$ are in general nonlinear and represent the detector-recording mechanisms. Finally the term $\eta(x,y)$ represents the *additive noise* which has an image-dependent random component, that is $f[g(w)]\eta_1$ and an image independent random component η_2.
This noise model leads to a possible classification of noise: additive noise (image independent) and multiplicative noise (image dependent).

Moreover the noise affecting the image, can have various distributions; the most common are: gaussian, uniform, and impulsive one. In this paper we will consider above all the gaussian case.

3. ESTIMATION OF THE NOISE

Many papers in literature present algorithms to estimate the features of additive noise in digital images [3] [4] [5] [6]. Some of them [3] [5] are based on the idea to extract these parameters starting from a small window of the image in which the original distribution of the pixel values can be supposed uniform enough. In this case the original image histogram would be made up of only one spectral line in $\bar{x}$, where $\bar{x}$ is the value of all pixels in the window. In this case a gaussian additive noise with zero mean produces a gaussian distribution histogram with the mean value $\bar{x}$ and a variance equal to noise variance. The main idea is to give more importance to those windows having a spectral distribution more similar to a gaussian one. The χ^2 is the mathematical parameter that gives a measure of that similarity [7] [8].
Let us have an expected gaussian distribution defined in an interval coincident with the range of the values of image pixels. Now we can divide this range in N_I subintervals. If E_k is the expected number of samples in the k^{th} interval and O_k is the real number of samples falling in the same interval, then we can compute χ^2 as:

$$\chi^2 = \sum_{k=1}^{N_I} \frac{(O_k - E_k)^2}{E_k}.$$

The more this value is small, the more the sample distribution is similar to the theoretical one. If $\chi^2 \to 0$ then the agreement is good. The number of intervals N_I has not to be too small, because otherwise the difference between the two distributions could not be appreciated. However, it cannot be too large, because this number has to be very smaller than the number of the samples.
Regarding the interval subdivision criterion we can choose between two alternatives.
The first method is a linear subdivision with respect to the x-axis. For example choosing as break points $\pm\sigma$, $\pm 2\sigma$, etc.
The second one, preferred in the following, considers a subdivision based on equiprobable intervals. With the last approach better results are obtained, because in every interval a significant number of samples falls.
The interval number N_I has been chosen equal to 8 and then the break points are:
$\Pi_{1,2,3} = \pm 0.3186\,\sigma, \pm 0.6745\,\sigma, \pm 1.1503\,\sigma.$

Figure 1 shows the graph of the probability distribution $P(x)$ of a gaussian noise. In the same graph the error function $\Phi(x)$, corresponding to an $erf(x)$ function, is also shown.

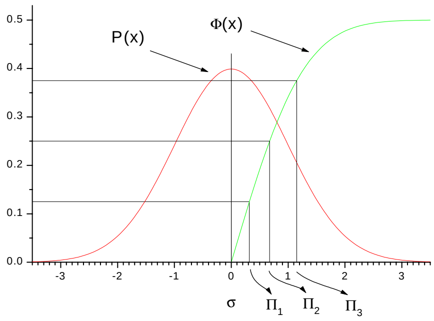

Fig. 1. Interval subdivision

Among the uniform zones in the image, more importance has to be given to those in which the mean value $\bar{x}$ of the pixels is close to the central value. Otherwise the final distribution of the noisy image could have cut tails. This effect can be considered through another parameter, that is the percentage of the saturated pixels p_{01}. This value has to be as less as possible. Let us consider that in a standard noise free image, the number of saturated pixels is very small and then a gaussian noise with a small variance would not substantially modify this value.

4. THE FUZZY SYSTEM FOR NOISE ESTIMATION

The idea is to obtain the estimation of the noise variance starting from a certain number of parameters easily obtained from the processed image by a preprocessor PP. As shown in section 3, these parameters are: χ^2, $\bar{x}$, and p_{01}.

Figure 2 shows, for a typical test image, the simulation results that prove how the goodness of noise estimation is linked to the three parameters χ^2, $\bar{x}$, and p_{01} that we use. From the simulations, we can notice that this particular shape for the errors distribution is maintained in every image and for every noise type.

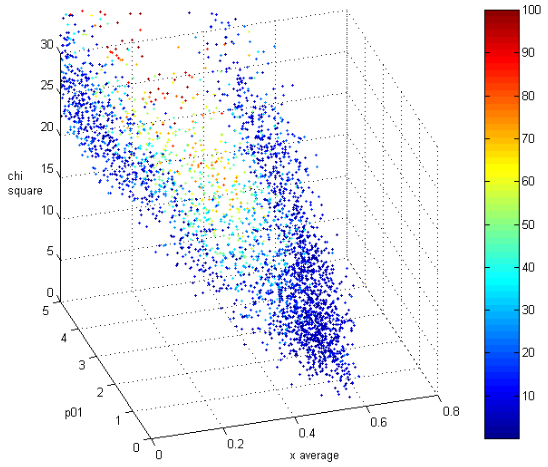

Fig. 2. Errors on noise estimation relating to the parameters χ^2, $\bar{x}$, and p_{01}

Figure 3 shows the same results of the previous simulation, but the data are mapped on the original picture. The darker zones correspond to better variance estimation.

Fig. 3. Sigma estimation relating to picture Lena

Figure 4 shows how the parameters χ^2, p_{01}, and $\bar{x}$ change into the same image. From the analysis of these trends we can see the correlation between the parameters and noise estimation into the image. The darker zones correspond to higher values of the parameter.

Using the values of these parameters we can weight the goodness of σ_i computed in the i^{th} window.
For this purpose a Sugeno fuzzy system $FS1$ which gives in output a value $h_i \in [0, 1]$, has been developed. The values σ_i and h_i are computed on non-overlapping N_W windows covering the whole image. These two parameters are the inputs of another fuzzy system $FS2$ which calculates the final value σ as follows:

$$\sigma = \frac{\sum_{i=1}^{N_W} h_i \cdot \sigma_i}{\sum_{i=1}^{N_W} h_i}.$$

Figure 5 shows the block diagram of the whole estimation system.

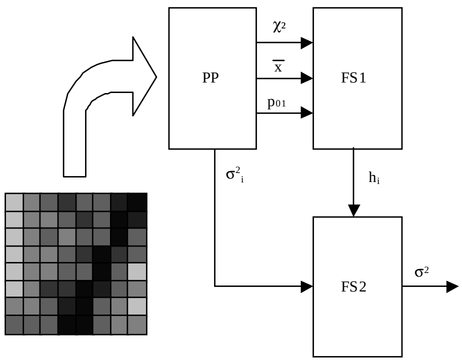

Fig. 5. The fuzzy processing flow

The shape of the input membership functions has been chosen triangular and the overlap degree equals two. In figure 6 the MF definitions for χ^2, p_{01}, and $\bar{x}$ are shown.

Moreover the value of χ^2 depends on the number of intervals N_I. In fact [7] the mean value of a χ^2 distribution tends to $N_I - 1$ if the samples number $N_S \to \infty$. The MFs definition for χ^2 must consider this property.

The fuzzy rules used in FS1 for the calculus of h_i are the following:

if (χ^2 is L) and (p_{01} is L) and ($\bar{x}$ is M) then (h_i is H)
if (χ^2 is L) and (p_{01} is L) and ($\bar{x}$ is H) then (h_i is H)
if (χ^2 is L) and (p_{01} is L) and ($\bar{x}$ is L) then (h_i is H)
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if (χ^2 is M) and (p_{01} is M) and ($\bar{x}$ is M) then (h_i is L)

where L, M, and H in the antecedents represent respectively the MFs Low, Medium, and High with respect to the input variables. In consequence, L and H are instead the singleton MFs Low and High, with respect to the output variable h_i, and their values are 0 and 1.

(a) Lena (b) Flowers

Fig. 7. Test images

The rules and the MFs parameters have been tuned taking into account the shape of a mean errors distribution, computed on various test images with different noise variances (one of these has been shown in figure 2).

5. RESULTS OF THE SIMULATIONS

The rules and the MF parameters have been tuned testing the system on different kind of images and with various window sizes. Note that the luminance values have been normalized to $[0, 1]$. The results refer to two images, shown in figure 7, often used in this kind of analysis (Lena and Flowers) for different values of the gaussian additive noise variance.

In tables 1 and 2 the results, in terms of relative errors, achieved for different values of the gaussian additive noise variance, are shown. Our values (SMR) are compared with those obtained implementing the algorithm in [4] (MJR). It can be noticed that our algorithm performance is very good also if the test image has not large uniform zones.

Table 1. Variance estimation results for Lena image

σ^2	SMR	MJR
0.04	5.25 %	13.75 %
0.03	0.33 %	6.67 %
0.02	7.50 %	4.00 %

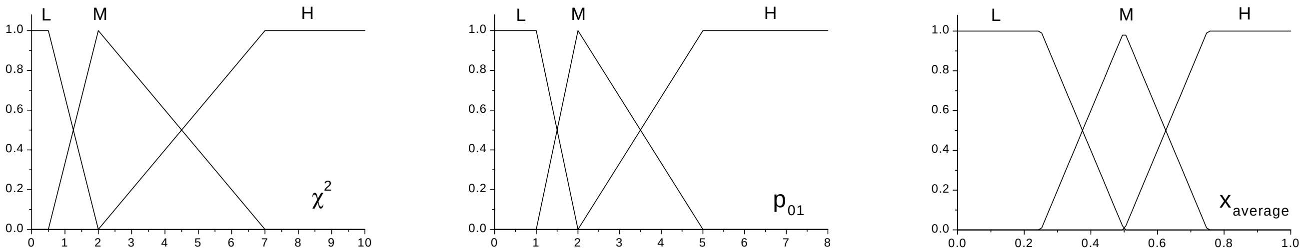
Table 2. Variance estimation results for Flowers image

σ^2	SMR	MJR
0.04	1.25 %	58.75 %
0.03	4.33 %	58.33 %
0.02	18.00 %	94.50 %

For the calculus of χ^2 we choose 8 intervals with the same probability and a window size of 8×8 pixel.

(a) Values of χ^2 (b) Values of p_{01} (c) Values of $\bar{x}$

Fig. 4. Noise estimation parameters mapped on picture Lena.



(a) MFs for χ^2 (b) MFs for p_{01} (c) MFs for $\bar{x}$

Fig. 6. Definition of the input MFs

6. CONCLUSIONS

The paper presents a new method to obtain a good estimation of the variance of a gaussian additive noise. It uses a fuzzy system that processes three parameters which can be easily extracted from the image. The results are better than the others obtained with other algorithms presented in literature. Moreover the methodology shown is easily applied to other cases with different noise distributions and it works also in cases in which many other approaches fail.

7. REFERENCES

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